AAPT UNITED STATES PHYSICS TEAM
AIP 2016

USA Physics Olympiad Exam

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Important Instructions for the Exam Supervisor

- This examination consists of two parts: Part A has four questions and is allowed 90 minutes; Part B has two questions and is allowed 90 minutes.
- The first page that follows is a cover sheet. Examinees may keep the cover sheet for both parts of the exam.
- The parts are then identified by the center header on each page. Examinees are only allowed to do one part at a time, and may not work on other parts, even if they have time remaining.
- Allow 90 minutes to complete Part A. Do not let students look at Part B. Collect the answers to Part A before allowing the examinee to begin Part B. Examinees are allowed a 10 to 15 minutes break between parts A and B.
- Allow 90 minutes to complete Part B. Do not let students go back to Part A.
- Ideally the test supervisor will divide the question paper into 4 parts: the cover sheet (page 2), Part A (pages 3-6), Part B (pages 8-9), and several answer sheets for one of the questions in Part A (pages 11-12). Examinees should be provided parts A and B individually, although they may keep the cover sheet. The answer sheets should be printed single sided!
- The supervisor *must* collect all examination questions, including the cover sheet, at the end of the exam, as well as any scratch paper used by the examinees. Examinees may *not* take the exam questions. The examination questions may be returned to the students after April 15, 2016.
- Examinees are allowed calculators, but they may not use symbolic math, programming, or graphic features of these calculators. Calculators may not be shared and their memory must be cleared of data and programs. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. Examinees may not use any tables, books, or collections of formulas.

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INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems. Each question is worth 25 points. Do not look at Part B during this time.
- After you have completed Part A you may take a break.
- Then work Part B. You have 90 minutes to complete both problems. Each question is worth 50 points. Do not look at Part A during this time.
- Show all your work. Partial credit will be given. Do not write on the back of any page. Do not write anything that you wish graded on the question sheets.
- Start each question on a new sheet of paper. Put your AAPT ID number, your name, the question number and the page number/total pages for this problem, in the upper right hand corner of each page. For example,

  AAPT ID #
  Doe, Jamie
  A1 - 1/3

- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers/solutions) on this contest until after April 15, 2016.**

Possibly Useful Information. You may use this sheet for both parts of the exam.

$g = 9.8\ \mathrm{N/kg}$	$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
$k = 1/4\pi\epsilon_0 = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$	$k_\mathrm{m} = \mu_0/4\pi = 10^{-7}\ \mathrm{T \cdot m/A}$
$c = 3.00 \times 10^8\ \mathrm{m/s}$	$k_\mathrm{B} = 1.38 \times 10^{-23}\ \mathrm{J/K}$
$N_\mathrm{A} = 6.02 \times 10^{23}\ (\mathrm{mol})^{-1}$	$R = N_\mathrm{A} k_\mathrm{B} = 8.31\ \mathrm{J/(mol \cdot K)}$
$\sigma = 5.67 \times 10^{-8}\ \mathrm{J/(s \cdot m^2 \cdot K^4)}$	$e = 1.602 \times 10^{-19}\ \mathrm{C}$
$1\mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$	$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s} = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}$
$m_e = 9.109 \times 10^{-31}\ \mathrm{kg} = 0.511\ \mathrm{MeV/c^2}$	$(1+x)^n \approx 1 + nx$ for $\lvert x \rvert \ll 1$
$\sin\theta \approx \theta - \frac{1}{6}\theta^3$ for $\lvert\theta\rvert \ll 1$	$\cos\theta \approx 1 - \frac{1}{2}\theta^2$ for $\lvert\theta\rvert \ll 1$

Part A

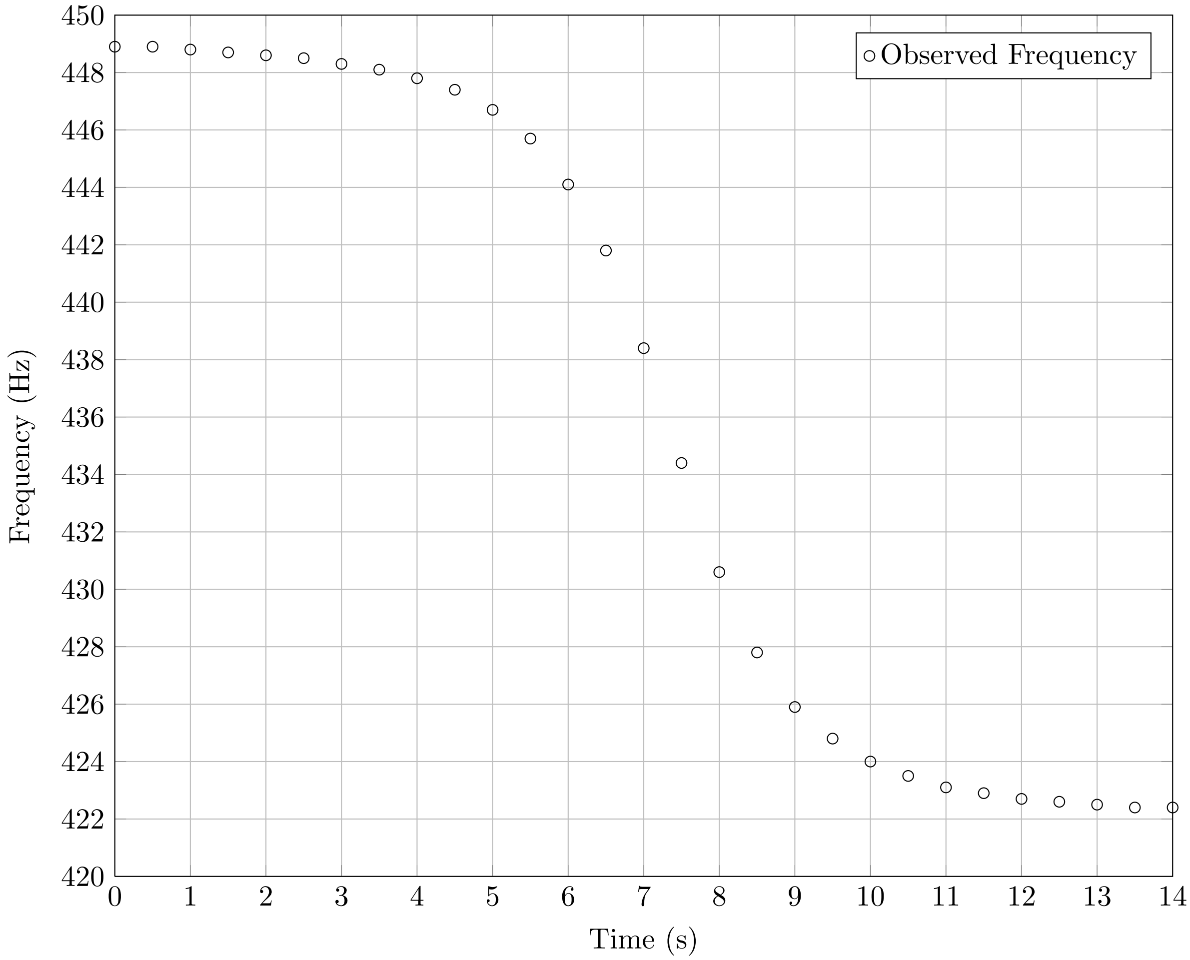
Question A1

The Doppler effect for a source moving relative to a stationary observer is described by

$$f = \frac{f_0}{1 - (v/c)\cos\theta}$$

where f is the frequency measured by the observer, f_0 is the frequency emitted by the source, v is the speed of the source, c is the wave speed, and θ is the angle between the source velocity and the line between the source and observer. (Thus $\theta = 0$ when the source is moving directly towards the observer and $\theta = \pi$ when moving directly away.)

A sound source of constant frequency travels at a constant velocity past an observer, and the observed frequency is plotted as a function of time:



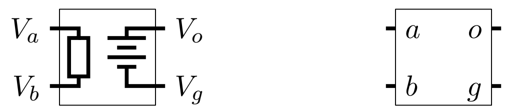
The experiment happens in room temperature air, so the speed of sound is 340 m/s.

a. What is the speed of the source?

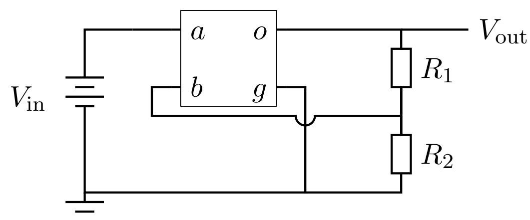
b. What is the smallest distance between the source and the observer?

Question A2

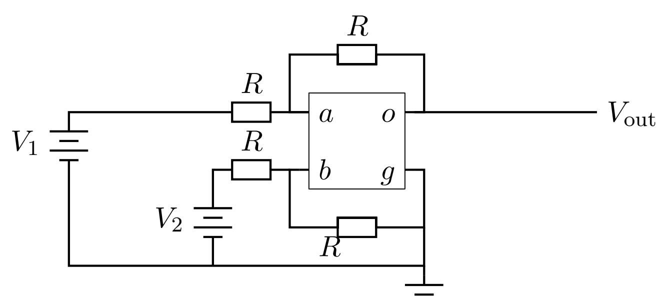
A student designs a simple integrated circuit device that has two inputs, V_a and V_b, and two outputs, V_o and V_g. The inputs are effectively connected internally to a single resistor with effectively infinite resistance. The outputs are effectively connected internally to a perfect source of emf $\mathcal{E}$. The integrated circuit is configured so that $\mathcal{E} = G(V_a - V_b)$, where G is a very large number somewhere between 10^7 and 10^9. The circuits below are chosen so that the precise value of G is unimportant. On the left is an internal schematic for the device; on the right is the symbol that is used in circuit diagrams.



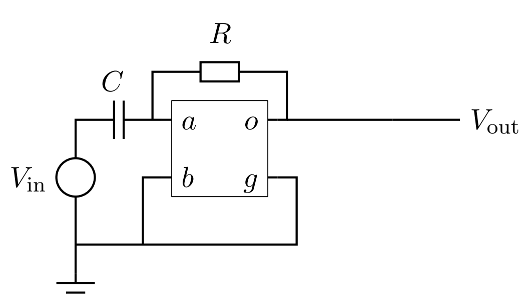
a. Consider the following circuit. $R_1 = 8.2\ k\Omega$ and $R_2 = 560\ \Omega$ are two resistors. Terminal g and the negative side of V_{in} are connected to ground, so both are at a potential of 0 volts. Determine the ratio $V_{\text{out}}/V_{\text{in}}$.



b. Consider the following circuit. All four resistors have identical resistance R. Determine V_{out} in terms of any or all of V_1, V_2, and R.



c. Consider the following circuit. The circuit has a capacitor C and a resistor R with time constant $RC = \tau$. The source on the left provides variable, but bounded voltage. Assume V_{in} is a function of time. Determine V_{out} as a function of V_{in}, and any or all of time t and τ.



Question A3

Throughout this problem the inertial rest frame of the rod will be referred to as the rod's frame, while the inertial frame of the cylinder will be referred to as the cylinder's frame.

A rod is traveling at a constant speed of $v = \frac{4}{5}c$ to the right relative to a hollow cylinder. The rod passes through the cylinder, and then out the other side. The left end of the rod aligns with the left end of the cylinder at time $t = 0$ and $x = 0$ in the **cylinder's frame** and time $t' = 0$ and $x' = 0$ in the **rod's frame**.

The left end of the rod aligns with the left end of the cylinder at the same time as the right end of the rod aligns with the right end of the cylinder in the **cylinder's frame**; in this reference frame the length of the cylinder is 15 m.

For the following, sketch accurate, scale diagrams of the motions of the ends of the rod and the cylinder on the graphs provided. The horizontal axis corresponds to x, the vertical axis corresponds to ct, where c is the speed of light. Both the vertical and horizontal gridlines have 5.0 meter spacing.

a. Sketch the world lines of the left end of the rod (RL), left end of the cylinder (CL), right end of the rod (RR), and right end of the cylinder (CR) in the **cylinder's frame**.

b. Do the same in the **rod's frame**.

c. On **both** diagrams clearly indicate the following four events by the letters A, B, C, and D.

   A: The left end of the rod is at the same point as the left end of the cylinder

   B: The right end of the rod is at the same point as the right end of the cylinder

   C: The left end of the rod is at the same point as the right end of the cylinder

   D: The right end of the rod is at the same point as the left end of the cylinder

d. At event B a small particle P is emitted that travels to the left at a constant speed $v_P = \frac{4}{5}c$ in the **cylinder's frame**.

   i. Sketch the world line of P in the **cylinder's frame.**

   ii. Sketch the world line of P in the **rod's frame.**

e. Now consider the following in the **cylinder's frame**. The right end of the rod stops instantaneously at event B and emits a flash of light, and the left end of the rod stops instantaneously when the light reaches it. Determine the final length of the rod after it has stopped. You can assume the rod compresses uniformly with no other deformation.

Any computation that you do must be shown on a separate sheet of paper, and not on the graphs. Graphical work that does not have supporting computation might not receive full credit.

Question A4

The flow of heat through a material can be described via the *thermal conductivity* κ. If the two faces of a slab of material with thermal conductivity κ, area A, and thickness d are held at temperatures differing by ΔT, the thermal power P transferred through the slab is

$$P = \frac{\kappa A \Delta T}{d}$$

A large, flat lake in the upper Midwest has a uniform depth of 5.0 meters of water that is covered by a uniform layer of 1.0 cm of ice. Cold air has moved into the region so that the upper surface of the ice is now maintained at a constant temperature of $-10\ ^\circ\mathrm{C}$ by the cold air (an infinitely large constant temperature heat sink). The bottom of the lake remains at a fixed $4.0\ ^\circ\mathrm{C}$ because of contact with the earth (an infinitely large constant temperature heat source). It is reasonable to assume that heat flow is only in the vertical direction and that there is no convective motion in the water.

a. Determine the initial rate of change in ice thickness.

b. Assuming the air stays at the same temperature for a long time, find the equilibrium thickness of the ice.

c. Explain why convective motion can be ignored in the water.

Some important quantities for this problem:

Specific heat capacity of water	C_{water}	$4200\ \mathrm{J/(kg \cdot C^\circ)}$
Specific heat capacity of ice	C_{ice}	$2100\ \mathrm{J/(kg \cdot C^\circ)}$
Thermal conductivity of water	κ_{water}	$0.57\ \mathrm{W/(m \cdot C^\circ)}$
Thermal conductivity of ice	κ_{ice}	$2.2\ \mathrm{W/(m \cdot C^\circ)}$
Latent heat of fusion for water	L_{f}	$330,000\ \mathrm{J/kg}$
Density of water	ρ_{water}	$999\ \mathrm{kg/m^3}$
Density of ice	ρ_{ice}	$920\ \mathrm{kg/m^3}$

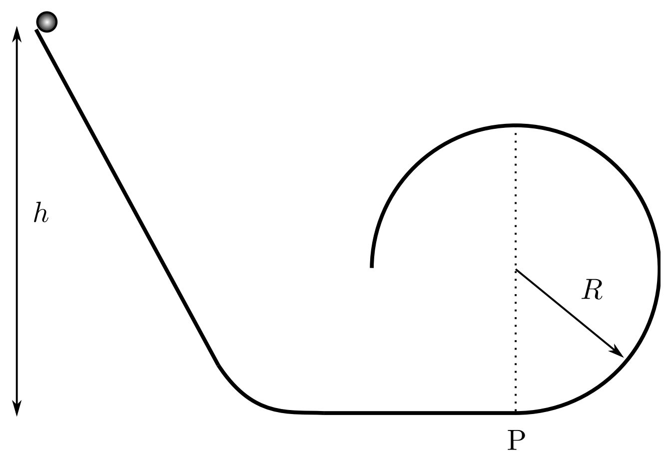
STOP: Do Not Continue to Part B

If there is still time remaining for Part A, you should review your work for Part A, but do not continue to Part B until instructed by your exam supervisor.

Part B

Question B1

A uniform solid spherical ball starts from rest on a loop-the-loop track. It rolls without slipping along the track. However, it does not have enough speed to make it to the top of the loop. From what height h would the ball need to start in order to land at point P directly underneath the top of the loop? Express your answer in terms of R, the radius of the loop. Assume that the radius of the ball is very small compared to the radius of the loop, and that there are no energy losses due to friction.



Question B2

a. A spherical region of space of radius R has a uniform charge density and total charge $+Q$. An electron of charge $-e$ is free to move inside or outside the sphere, under the influence of the charge density alone. For this first part ignore radiation effects.

   i. Consider a circular orbit for the electron where $r < R$. Determine the period of the orbit T in terms of any or all of r, R, Q, e, and any necessary fundamental constants.

   ii. Consider a circular orbit for the electron where $r > R$. Determine the period of the orbit T in terms of any or all of r, R, Q, e, and any necessary fundamental constants.

   iii. Assume the electron starts at rest at $r = 2R$. Determine the speed of the electron when it passes through the center in terms of any or all of R, Q, e, and any necessary fundamental constants.

b. Accelerating charges radiate. The total power P radiated by charge q with acceleration a is given by

$$P = C\xi a^n$$

where C is a dimensionless numerical constant (which is equal to $1/6\pi$), ξ is a physical constant that is a function only of the charge q, the speed of light c, and the permittivity of free space ϵ_0, and n is a dimensionless constant. Determine ξ and n.

c. Consider the electron in the first part, except now take into account radiation. Assume that the orbit remains circular and the orbital radius r changes by an amount $|\Delta r| \ll r$.

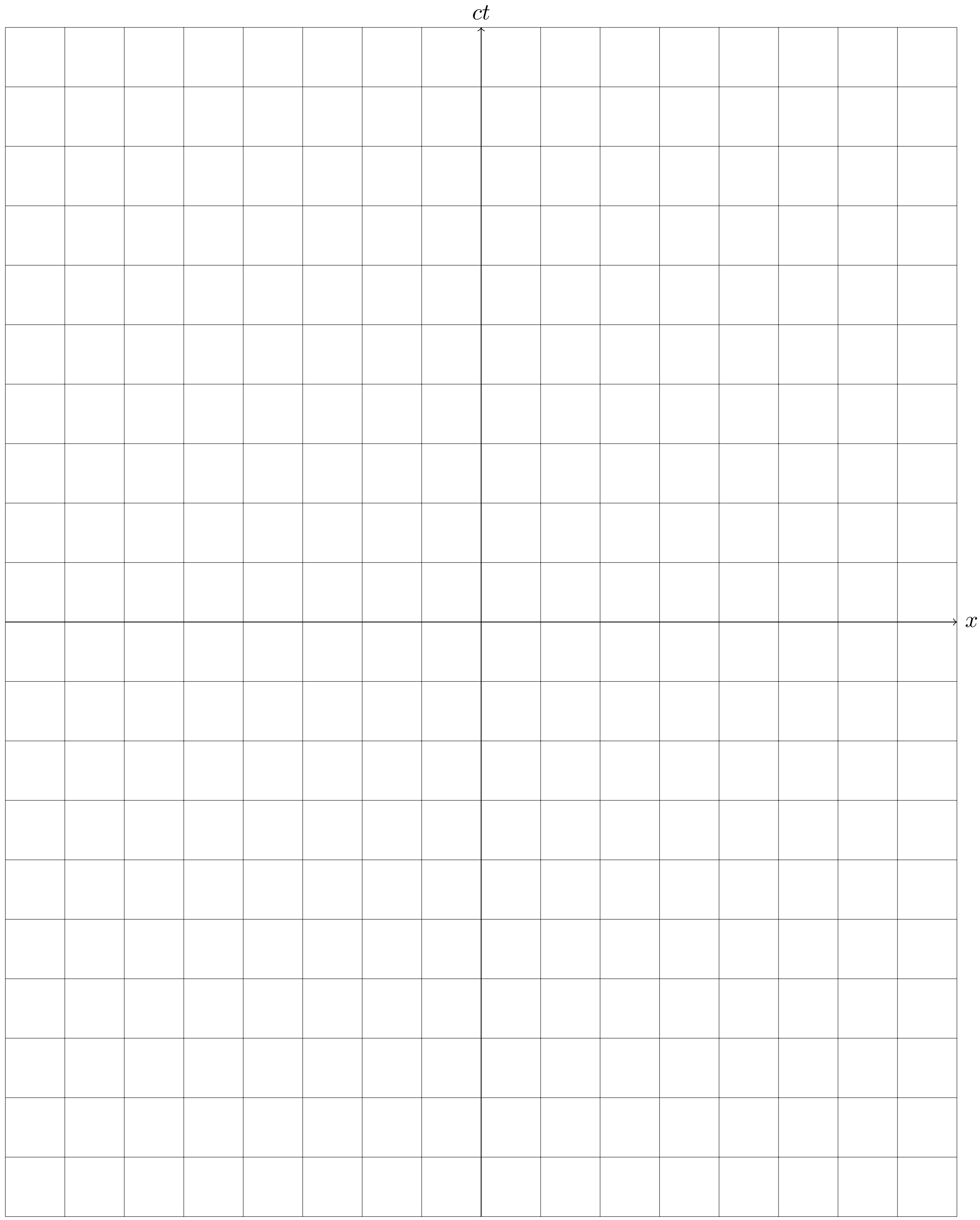
   i. Consider a circular orbit for the electron where $r < R$. Determine the change in the orbital radius Δr during one orbit in terms of any or all of r, R, Q, e, and any necessary fundamental constants. Be very specific about the sign of Δr.

   ii. Consider a circular orbit for the electron where $r > R$. Determine the change in the orbital radius Δr during one orbit in terms of any or all r, R, Q, e, and any necessary fundamental constants. Be very specific about the sign of Δr.

Answer Sheets

Following are answer sheets for some of the graphical portions of the test.

The Cylinder's Frame



The Rod's Frame