Snell's Law

Part A: Light Propagation Through a Semi-Sphere

A.1
[0.5pt]

Using Snell's law, larger angle of refraction corresponds to larger refractive index that the light sees in the semi-sphere (n_1). [0.2pt]

This means that the colour with a smaller angle of refraction propagates faster in the semi-sphere, which is colour *a*. [0.3pt]

A.2
[1.0pt]

The light rays no longer exit the bottom of semi-sphere when the ray is refracted by 90^{o} with respect to the normal. [0.2pt]

Using Snell's law,

$$n_x sin\theta_i = n_2 \sin(90°)$$ [0.2pt]

Replace "x" with "a" and "b" for the condition of ray a and ray b, respectively.

For ray a, $$n_a sin(50°) = 1.0 \sin(90°)$$
$$n_a = 1.305$$ [0.2pt]

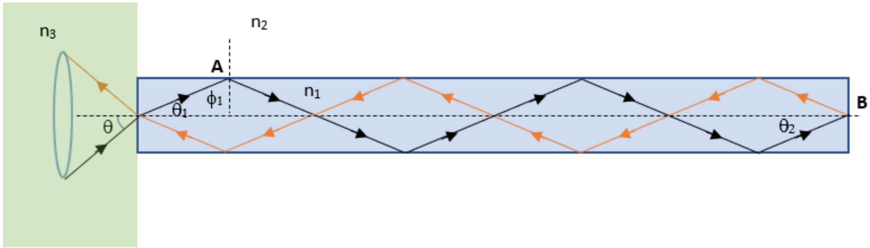
For ray b, $$n_b sin(45°) = 1.0 \sin(90°)$$
$$n_b = 1.414$$ [0.2pt]

Difference in refractive index, Δn = **0.109** [0.2pt]

Part B: Light Propagation Through a Cylindrical Rod

B.1
[2.0pt]

Given $n_1 = 1.5$, $n_2 = 1.0$, $n_3 = 1.4$,



Identify 2 conditions that need to be obeyed: [0.2pt]

#1: Light incident at point A should be totally reflected.

$$n_1 sin\phi_1 > n_2 \sin(90°)$$

[0.3pt]

$$\phi_1 > sin^{-1}\frac{1.0}{1.5} = 41.81°$$

The condition for angle $\theta_1 < 90° - 41.81° = 48.19°$ [0.2pt]

Using Snell's law,

$$\theta < sin^{-1}\frac{1.5}{1.4}\sin(48.19°) = 53°$$

[0.2pt]

B.1 **[cont.]**	**#2: Light incident at point B should be totally reflected.**	
	$$n_1 sin\theta_2 > n_2 \sin{(90°)}$$	
	$$\theta_2 > sin^{-1}\frac{1.0}{1.5} = 41.81°$$	[0.3pt]
	The condition for angle $q_1 > 41.81^o$	[0.2pt]
	Using Snell's Law	
	$$\theta > sin^{-1}\frac{1.5}{1.4}\sin{(41.81°)} = 45.58°$$	[0.2pt]
	Therefore, the incident angle, q, where light is totally reflected back to the polymer is $45.58^o < q < 53^o$.	[0.4pt]

B.2 (i) [0.6pt]	When right end of rod is coated with oil with n_4 = 1.6, condition #2 changes.	[0.2pt]
	Light incident on point B will always be refracted.	[0.2pt]
	Light CANNOT be totally reflected back to the polymer.	[0.2pt]

B.2 (ii) **[0.9pt]**

When the setup is placed in water, $\mathbf{n_2 = 1.33}$. Both condition #1 and #2 change. [0.2pt]

#1:

$$\phi_1 > sin^{-1}\frac{1.33}{1.5} = 62.46°$$ [0.2pt]

$\theta_1 < 90° - 62.46° = 27.54°$

Using Snell's law,

$$\theta < sin^{-1}\frac{1.5}{1.4}\sin{(27.54°)} = 29.7°$$

#2:

$$\theta_2 > sin^{-1}\frac{1.33}{1.5} = 62.46°$$

Using Snell's law,

$$\theta > sin^{-1}\frac{1.5}{1.4}\sin{(62.46°)} = 71.81°$$ [0.2pt]

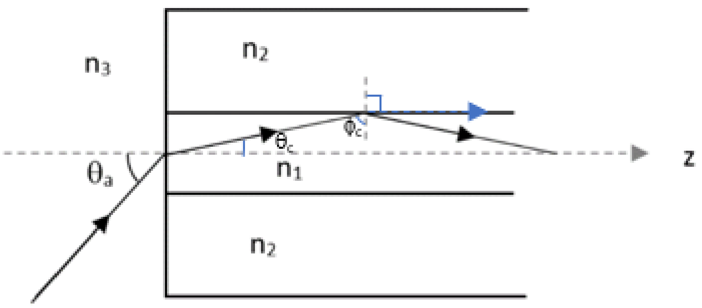
As conditions #1 and #2 does not overlap. [0.1pt]

Again, light CANNOT be totally reflected back to the polymer. [0.2pt]

Part C: Light Propagation Through an Optical Fibre

C.1

[2.0pt]



In order for light to not refractive out from the fiber, the angle of refraction that the light makes with the normal of the fiber core/cladding boundary needs to be 90°. Using Snell's law,

$$n_1 sin\phi_c = n_2 \sin(90°) \quad \text{..... (1)}$$ [0.4pt]

Taking the square of eq. (1), we have,

$$n_1^2 sin^2\phi_c = n_2^2$$

[0.2pt]

Using the trigonometry identity, we have,

$$n_1^2(1 - cos^2\phi_c) = n_2^2$$

Rearranging, we got,

C.1

[cont.]

$$n_1^2 cos^2\phi_c = n_1^2 - n_2^2$$

Or, [0.2pt]

$$n_1^2 sin^2\theta_c = n_1^2 - n_2^2 \quad ....(2)$$

Taking the square root of eq. (2), we have,

$$n_1 sin\theta_c = \sqrt{n_1^2 - n_2^2}$$ [0.4pt]

With this, we can use Snell's law to calculate the maximum angle of light incident,

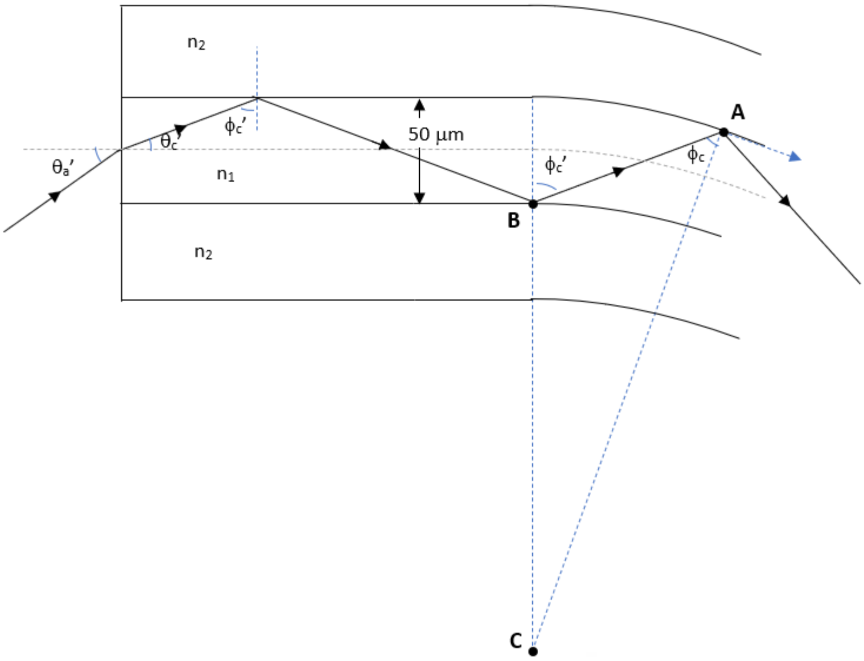
$$n_3 sin\theta_a = n_1 sin\theta_c = \sqrt{n_1^2 - n_2^2}$$ [0.4pt]

$$\theta_a = sin^{-1}\frac{\sqrt{n_1^2 - n_2^2}}{n_3}$$ [0.4pt]

or $\theta_a = \sin^{-1}\sqrt{{n_1}^2 - {n_2}^2}$

C.2

[2.6pt]



- Identifying the correct relationship between $\phi_c$and $\phi\prime_c$ [0.4pt]

- Find the relationship between ϕ_c and ϕ_c':

- Using the Law of Sine,

$$sin\phi_c = \frac{CB}{CA} sin(\pi - \phi_c') \quad [0.4pt]$$

- For light guiding at point A,

$\sin \phi_c = \frac{n_2}{n_1}$ [0.4pt]

CB = 10000-25 = 9975 μm

CA = 10000+25 = 10025 μm [0.4pt]

C.2

[cont.]

$$sin(\pi - \phi_c{}') = \sin(\phi_c')$$

Therefore,

$$\sin(\phi_c') = \frac{10025n_2}{9975n_1}$$

[0.4pt]

Rearrange into eq.(1) and derive following part (a):

$$a.n_1\sin(\phi_c') = n_2, \qquad where\ a = \frac{9975}{10025}$$

$$a^2\,n_1^2\sin^2(\phi_c') = n_2^2$$

$$a^2n_1^2(1 - cos^2\phi'_c) = n_2^2$$

[0.4pt]

$$a^2n_1^2cos^2\phi'_c = a^2n_1^2 - n_2^2$$

$$a^2n_1^2sin^2\theta'_c = a^2n_1^2 - n_2^2$$

$$n_1sin\theta'_c = \frac{1}{a}\sqrt{a^2n_1^2 - n_2^2}$$

To calculate the new angle of acceptance,

$$n_3 sin\theta'_a = n_1 sin\theta'_c = \frac{1}{a}\sqrt{a^2n_1^2 - n_2^2}$$

$$\theta'_a = sin^{-1}\left(\frac{10025}{9975n_3}\sqrt{a^2n_1^2 - n_2^2}\right)$$

[0.2pt]

C.3

[0.4pt]

For case C.1: State that $n_3 = 1$;

[0.1pt]

$$\sqrt{n_1^2 - n_2^2} = \sqrt{1.45^2 - 1.44^2} = 0.17$$

$$\theta_a = sin^{-1} 0.17 = 9.79^0$$ [0.1pt]

For case C.2: Substitute,

$$\theta'_a = sin^{-1}\left(\frac{10025}{9975}\sqrt{\left(\frac{9975}{10025}\right)^2 1.45^2 - 1.44^2}\right)$$ [0.1pt]

$$= 5.15°$$ [0.1pt]