**A.1**

**[1.3pt]**

The moon completes one orbit in about a month, or more quantitatively, moves through **360 degrees in 27.3 days**. 360 divided by 27.3 = 13.19 degrees per day. Each day the moon will appear about **13deg** to the **east** of its position at the same time on the previous day.

Now convert to motion per hour. 13.19 degrees/day divided by 24 hrs/day = 0.55 degrees/hr. The moon passes through a little more than ½ of a degree in one hour of time. [0.5pt]

Since 1deg = 60 arcminutes, then 0.55deg = 33 arcminutes (this is a ratio, 1:60 as 0.55:33). Therefore the moon moves through 33 arcminutes per hour of time.

Since 1 arcminute = 60 arcseconds, then the moon moves through 33 x 60 arcseconds in a hour = 1978 arcsec per hour of time.

Moving 0.55 degrees / hr divided by 60 minutes per hr = 0.0092 degrees / min. The moon moves through a small fraction of a degree in one minute of time.

As 1deg = 60 arcminutes, then 0.0092deg = 0.55 arcminutes. The moon moves through 0.55 arcminutes per minute of time. [0.4pt]

Since 1 arcminute = 60 arcseconds, the moon moves through 33 arcseconds per minute of time.

Moving 0.55 degrees per hour divided by 3600 seconds per hour = 0.000153 degrees in a second of time.

If the moon moves 0.55 arcminute per minute of time (from part b) then the moon will move 0.55 / 60 = 0.0092 arcminutes in one second of time. [0.4pt]

0.0092 arcminutes X 60 arcseconds per arcminute = 0.55deg per second of time.

**A.2**

**[0.5pt]**

The diameter of the moon = 30 arcminutes = 0.5 degree (given in the notes).

From part (b), the moon moves through 0.55 arcminutes per minute of time

The time it takes to move a specific distance = that distance divided by the rate of motion. The time it take the Moon to move one lunar diameter = distance traveled (30 arcminutes) / divided by rate (0.55 arcminutes per minute) = 54.5 minutes, just under an hour. [0.5pt]

## Part B: Using Lunar Occultations to Precisely Determine Radio Source Positions: The Case of 3C 273

| | | |
|---|---|---|
| **B.1** **[0.6pt]** | The correct answer is: A. The diffraction pattern of the telescope. | [0.6pt] |

| | | |
|---|---|---|
| **B.2** **[0.6pt]** | The correct answer is: C. Both components are aligned with the limb of the Moon when they reappear. | [0.6pt] |

| | | |
|---|---|---|
| **B.3** **[0.6pt]** | Since it was calculated that the moon travels at 33 arcsec per minute w.r.t the background stars, and the panel shows about 1/2 minute between the components A and B, this suggests that these two components are separated about 15 arcsec. (any number within this estimation shall be considered valid as long as it is properly explained). | [0.6pt] |

## Part C: The Breakthrough Discovery of 3C 273's True Nature

**C.1**
**[0.6pt]**

The student can check the figure to estimate the wavelengths by eye where certain emission lines are happening. The reasoning shall be written. For example, one could say that the H-gamma line for 3C 273 is located almost at 500nm, so maybe 490 or 495 nm would be a good estimate. For the comparison spectrum, since it is closer to 400nm, maybe the H-gamma of the comparison spectrum maybe be around 440nm (actually, 434nm). [0.3pt]

The resulting redshift of 3C273 is $(\lambda-\lambda_0)/\lambda_0 \sim 0.158$. Other results that are similar to this one will be also considered correct as long as the wavelengths are reasonably estimated. [0.3pt]

**C.2**
**[0.6pt]**

Solving for the mass, we have that $M=(c^2r/2G)[1-1/(1+z)^2]$. [0.2pt]

If we put it at the edge of the Milky Way (at 100 kpc), then $r\sim 3\times 10^{21}$ m and $M\sim 10^{17}M_\odot$ (larger than the entire mass of the Milky Way!) [0.2pt]

if we put it at the edge of the Solar System (at 100 au), then $r\sim 1.5\times 10^{13}$ m and $M\sim 10^{9}M_\odot$ . This would totally disrupt the Solar System. [0.2pt]

| | | |
|---|---|---|
| **C.3** **[0.6pt]** | Since $z \sim v/c$ for small redshift and Hubble's law is $v = Hd$, we can combine these two equations to obtain $d = zc/H$. | [0.3pt] |
| | If we use that H=75 km/s/Mpc, then d = 0.158*300 000 / 75 ~ 632 Mpc. (The current estimation is that the distance is 749 Mpc). This is thousands of times farther away than the size of the Milky Way, and must therefore, be an independent object very much farther away. | [0.3pt] |

## Part D: The Intrinsic Luminosity of the Radio Source 3C 273

**D.1**
**[0.6pt]**

If we use the distance of 632 Mpc that has been obtained before, that gives us $d = 2 \times 10^{25}$m. Therefore, $S_\nu = 2.5 \times 10^{-22}\nu^{-0.3}$.

The Luminosity is $L = 4\pi d^2 S_\nu = 4\pi\left(2 \times 10^{25}\right)^2 \nu^{-0.3} = 1.25 \times 10^{30}\ \nu^{-0.3}\ \mathrm{W\ m^{-2}\ Hz^{-1}}$.

[0.6pt]

**D.2**
**[0.6pt]**

The total radio luminosity can be calculated by integrating for all frequencies:

$$L = \int_{10^7}^{10^{11}} L_\nu \,\mathrm{d}\,\nu = \left(\frac{1.25\times10^{30}}{0.7}\right)\nu^{0.7}\Big|_{\nu=10^7}^{\nu=10^{11}} = 9 \times 10^{37}\ \mathrm{W}$$

[0.6pt]

**D.3**
**[0.6pt]**

Comparing with the Sun, $\frac{L}{L_{\text{sun}}} = \frac{9\times10^{37}}{3.82\times10^{26}} \sim 2.3 \times 10^{11}$. It's almost a trillion times brighter, and brighter than our Milky Way.

[0.6pt]

## Part E: The Power Source of 3C 273

| | | |
|---|---|---|
| **E.1** **[0.7pt]** | We are going to be using the mass-energy expression. Electrons have a mass-energy of 0.5 MeV (other units; e.g, $8 \times 10^{14}$J will also be considered valid). | [0.4pt] |
| | This energy, converted into wavelength, goes into 0.000002 nm. (other units, or solution expressed as frequency e.g, $1.5 \times 10^{14}$ GHz will also be considered valid). This is well within the X-rays. | [0.3pt] |

| | | |
|---|---|---|
| **E.2** **[0.7pt]** | The gravitational potential energy of a mass m is given by W=-GMm/r. Since the smallest radius the particle reaches is the Schwarzschild radius, W = -GMmc$^2$/2GM = -mc$^2$/2. Therefore, one solar mass provides ~ $1.8 \times 10^{47}$J/yr ~$5.7 \times 10^{39}$ J/s. This is still an order of magnitude more than what is needed to power 3C273, so it even leaves room for a certain efficiency. | [0.7pt] |

## Part F: Modern Observations and the Nature of 3C 273's Components

**F.1**
**[0.7pt]**

To calculate the minimum we have to derive and equal to zero

Therefore,

$$\frac{\mathrm{d}U}{\mathrm{d}B} = 0 \rightarrow \frac{\mathrm{d}U_e}{\mathrm{d}B} + \frac{\mathrm{d}U_b}{\mathrm{d}B} = 0$$ [0.2pt]

Doing each of the terms separately,

$$\frac{\mathrm{d}U_e}{\mathrm{d}B} = -\frac{3}{2}B^{-5/2} = -\frac{3}{2}\frac{U_e}{B}$$

$$\frac{\mathrm{d}U_b}{\mathrm{d}B} = 2B = \frac{2U_b}{B}$$

Therefore, $\frac{\mathrm{d}U}{\mathrm{d}B} = -\frac{3}{2}\frac{U_e}{B} + \frac{2U_b}{B} = 0 \rightarrow \frac{U_e}{U_b} = \frac{4}{3}$ [0.5pt]

| | | |
|---|---|---|
| **F.2** **[0.7pt]** | The total energy, from the calculations in terms of the magnetic energy is $U = \frac{7}{3}U_b$. | |
| | Therefore, If $U = \frac{1}{2} \times 10^{38}$ J, then the magnetic energy will be $U_b = \frac{3}{14} \times 10^{38}$ J. | [0.3pt] |
| | For a volume of $10^{45}$ m$^3$, the energy density will be $u_b = \frac{\frac{3}{14}\times 10^{38}\text{ J}}{10^{45}\text{ m}^3} = 0.21 \times 10^{-7}$ J/m$^3$. | [0.2pt] |
| | Now, since $u = \frac{B^2}{2\mu}$, with $\mu$ = magnetic permeability = $4\pi \times 10^{-7}$ Tm/A, we have that $B = \sqrt{2 \times 4\pi \times 10^{-7} \times 0.21 \times 10^{-7}} \sim 2.3 \times 10^{-7}$ Tesla. | [0.2pt] |