QUESTION T1 [10 pts]. Three problems

This question consists of three independent problems.

T1-A [3.0 pts]. Hydrostatic gate

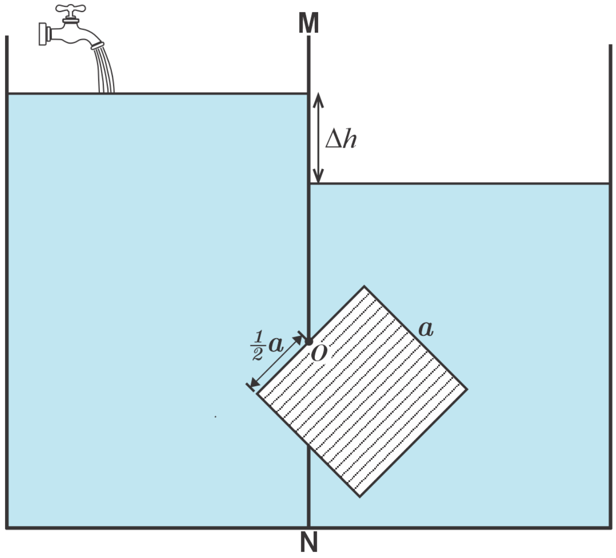

Fig. 1a.

Two water reservoirs are separated by a vertical wall MN, as shown in the figure 1a. The left reservoir can receive water from a source. It must be ensured that the maximum possible difference between the water levels is $\Delta h =$ 1.41 m. To achieve this, a slot of vertical size $a\sqrt{2}/2$ is cut in MN and sealed with a solid cubic block of side length a. The block has density $3\rho_0$, with ρ_0 the density of water. The block is fully submerged, fixed to the wall at O and can rotate without friction about the axis perpendicular to the plane of the figure that passes through point O.

T1-A1 [3.0 pts.] Calculate the value of a that ensures the maximum permissible difference in water levels is not exceeded.

T1-B [3.5 pts]. Electron-positron pair

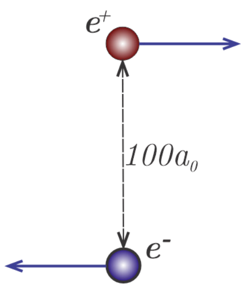

Fig. 1b.

At a certain instant of time, a positron e^+ is located at a distance of $100a_0$ from an electron e^-. The particles are moving in such a way that their velocities are antiparallel at that instant, perpendicular to their separation, as shown in figure 1b. It is known that each particle has an angular momentum of magnitude $\mu\hbar$ with respect to the system's center of mass, where μ is a dimensionless numerical factor. Assume that the only interaction between e^+ and e^- is electrostatic (both particles have the same mass m and equal magnitude of the charge but of opposite sign). Furthermore, assume that this system remains isolated and that its dynamics are classical and non-relativistic.

The Bohr radius a_0 is given by $a_0 = 4\pi\epsilon_0\hbar^2/(me^2)$.

T1-B1 [1.0 pts.] If $\mu = 4$, the system is bound, meaning that the particles move in a closed orbit around the system's center of mass. Find the maximum separation distance between e^+ and e^- in terms of a_0.

T1-B2 [2.5 pts]. If $\mu = \frac{15}{2}$, the system is unbound, meaing that the particles are no longer in closed orbit around the system's center of mass. Let $\vec{u}_\infty$ be the velocity of e^+ relative to e^- as the separation distance between them tends to infinity. Calculate the angle between $\vec{u}_\infty$ and the initial line of motion of e^+. Give your answer in degrees.

Hint 1: The eccentricty ε of a conic trajectory for the present case is given by

$$\varepsilon = \sqrt{1 + \frac{4L^2E}{k^2e^4m}}$$

where E and L are the total energy and the magnitude of the total angular momentum, respectively. k is Coulombs constant $k = 1/(4\pi\epsilon_0)$.

Hint 2: The equation of a conic in polar coordinates is

$$r = \frac{a}{1 - \varepsilon\cos(\theta)}$$

T1-C [3.5 pts]. Photodissociation of ozone

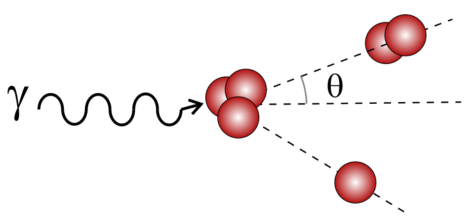

Fig. 1c.

A photon with angular frequency ω strikes an ozone molecule O_3 at rest, dissociating it (breaking it apart) into an oxygen molecule O_2 and an oxygen atom O, as shown in figure 1c. The photon is absorbed in this process. Let U_i and U_f be the ground state energies of the ozone and oxygen molecules, respectively. Assume that this system remains isolated and that its dynamics is classical (potential energies do not contribute to mass and the motion of the Oxygen atoms are non-relativistic). Use the relation $p = E/c$ for the linear momentum of the photon, where E is its energy.

T1-C1 [2.5 pts]. If the angle that the momentum of the outgoing O_2 with respect to the incident photon is θ, determine the minimum angular frequency $\omega_{\min}$ required for this dissociation to occur at this angle. Write your answer in terms of $\hbar$, c, θ, $\Delta U = U_f - U_i$, and the mass m of an oxygen atom.

T1-C2 [1.0 pts]. If $\theta = \pi/6$, calculate the value of $\hbar\omega_{\min} - \Delta U$. Give your answer in electronvolts (eV). Take $\Delta U = 1.10\ e$V and $m = 16.0$ amu.

Hint:

$$(1+x)^r = 1 + rx + \frac{r(r-1)}{2!}x^2 + \frac{r(r-1)(r-2)}{3!}x^3 + \cdots$$